تم الرفع بواسطة
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Linear

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Linear Algebra
First Exam
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27/40

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1 Hour

True / False. (1..0 points)

1. T ~~~~ $(A+B)^{-1} = B^{-1} + A^{-1}$.
2. T If A $is\ a$ 3×3 matrix, then there are two triangular matrices that factorizes A.
3. F If A $is\ a$ 3×5 and the system $Ax = b$ is consistent, then it has a unique solution.
4. T Elementary row operations never change the solution set to a linear system.
5. F The reduced row echelon form of a matrix is unique.
6. F If b is a linear combination of vectors a_1, a_2, and a_3 then the constants used in the linear combination are unique.
7. T If A, B and C are $n \times n$ nonsingular matrices then the system $(ABC)x = 0$ has only the trivial solution.
8. F The second row of AB is the second row of A multiplied on the right by the matrix B.
9. F If A is an invertible $n \times n$ matrix then the equation $Ax = b$ has exactly one solution for each vector b in R^n.
10. F The cofactor expansion of a matrix going down a column is the negative of the cofactor expansion of the same matrix going across a row.

5

2. If A is an 4×4 matrix, and the homogenous system $Ax = 0$ has only the trivial solution. Show that if $C = \alpha(A^T A)$, $\alpha \neq 0$, then C is nonsingular, then show that $de\,t(C) > 0$.

$Ax = 0 \Rightarrow A$ is nonsingular.

$c = \alpha(A^T A) \Rightarrow c$ is scalar multiply by a nonsingular matrix so c is nonsingular

$\det A^T = \det A$, $\det C = \alpha \det A^T \det A \Rightarrow \det C = \alpha \det^2 A$ why?

α is positive because negativ sign of α ~~change~~ make c singular $C = -\det A$!

$\therefore \det C = \alpha \underbrace{\det^2 A}_{\text{Positive}}$ Positive

$\therefore \det C$ is Positive $\Rightarrow > 0$

Page 1

3. Consider the matrix $A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 2 & 2 \\ 5 & 0 & 1 \end{bmatrix}$ (12 points)

i. Find *adjoint(A)* .

$$\operatorname{Adj}(A) = \begin{bmatrix} \begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & 2 \\ 5 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 5 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 0 \\ 5 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 5 & 0 \end{vmatrix} \\ \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 0 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} 2 & 11 & -10 \\ -1 & 3 & 5 \\ 2 & -6 & 7 \end{bmatrix}^T$$

$$\therefore \operatorname{Adj}(A) = \begin{bmatrix} 2 & 11 & -10 \\ -1 & 3 & 5 \\ 2 & -6 & 7 \end{bmatrix}^T$$

ii. Use cofactor expansion to find *det(A)*.

$\cancel{\det(A) = \det(\operatorname{Adj}(A))}$

$$\det(A) = 3\begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} - 1 * \begin{vmatrix} -1 & 2 \\ 5 & 1 \end{vmatrix} + 0 * \begin{vmatrix} -1 & 2 \\ 5 & 0 \end{vmatrix} = (3*2) + 11 = 6 + 11 = \underline{\underline{17}}$$

expansion about Row I

iii. Use your answers in parts (i) and (ii) to find the inverse of *A*.

$$A^{-1} = \frac{1}{\det(A)} * \operatorname{Adj} A \quad , \quad \det A = 17 \quad , \quad \operatorname{Adj} A = \begin{bmatrix} 2 & 11 & -10 \\ -1 & 3 & 5 \\ 2 & -6 & 7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{17} * \begin{bmatrix} 2 & 11 & -10 \\ -1 & 3 & 5 \\ 2 & -6 & 7 \end{bmatrix} = \begin{bmatrix} \frac{2}{17} & \frac{11}{17} & \frac{-10}{17} \\ \frac{-1}{17} & \frac{3}{17} & \frac{5}{17} \\ \frac{2}{17} & \frac{-6}{17} & \frac{7}{17} \end{bmatrix} \Rightarrow \therefore A^{-1} =$$

Page 2

4. Consider the matrix $A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 3 & 5 & 0 & 2 \\ -4 & 2 & 1 & 2 \\ 1 & 1 & 4 & 5 \end{bmatrix}$ (12 points)

i. Use row reduction to find the determinant of A. Once the matrix is in upper-triangular form (but not before) you may finish the determinant.

change to upper tringlur :-

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 3 & 5 & 0 & 2 \\ -4 & 2 & 1 & 2 \\ 1 & 1 & 4 & 5 \end{bmatrix} \xrightarrow[R_4=R_1]{R_1=R_4} \begin{bmatrix} 1 & 1 & 4 & 5 \\ 3 & 5 & 0 & 2 \\ -4 & 2 & 1 & 2 \\ 2 & 0 & 1 & 0 \end{bmatrix} \xrightarrow[R_2=R_2-3R_1,\ R_3=R_3+4R_1]{R_4=R_4-2R_1} \begin{bmatrix} 1 & 1 & 4 & 5 \\ 0 & 2 & -12 & -13 \\ 0 & 6 & 17 & 22 \\ 0 & -2 & -7 & -10 \end{bmatrix} \xrightarrow[R_4=R_4+R_2]{R_3=R_3-3R_2}$$

$$\begin{bmatrix} 1 & 1 & 4 & 5 \\ 0 & 2 & -12 & -13 \\ 0 & 0 & 53 & 61 \\ 0 & 0 & -19 & -23 \end{bmatrix} \xrightarrow{R_4=R_4+\frac{19}{53}R_3} \begin{bmatrix} 1 & 1 & 4 & 5 \\ 0 & 2 & -12 & -13 \\ 0 & 0 & 53 & 61 \\ 0 & 0 & 0 & (61\times\frac{19}{53})-23 \end{bmatrix}$$

$61 \times 19 = 1159$; $1159 \div 53 = 21\ldots$

$\therefore \frac{21.46}{53} - 23$

$21.84 - 23$

$= -1.15$

$$A = \begin{bmatrix} 1 & 1 & 4 & 5 \\ 0 & 2 & -12 & -13 \\ 0 & 0 & 53 & 61 \\ 0 & 0 & 0 & -1.15 \end{bmatrix}$$

$\therefore$ det (A) = multiple of elements on diagonal.

$= 1 * 2 * 53 * -1.15 = 106 * -1.15 = -121.5$

$\therefore \det(A) = -121.5$

ii. Determine whether A is singular or nonsingular? Why?

A is nonsingular because $\det(A) \neq 0$

iii. Use the matrix obtained in (i) to solve the system $Ax = [11 \quad 2 \quad 0 \quad 0]^T$

$$\left[\begin{array}{cccc|c} 1 & 1 & 4 & 5 & 11 \\ 0 & 2 & -12 & -13 & 2 \\ 0 & 0 & 53 & 61 & 0 \\ 0 & 0 & 0 & -1.15 & 0 \end{array}\right]$$

$\Rightarrow -1.15 X_4 = 0 \quad \therefore X_4 = 0$

$53 X_3 + 0 = 0 \quad \therefore X_3 = 0$

$2 X_2 - 0 - 0 = 2 \quad \therefore X_2 = 1$

$X_1 + 1 + 0 + 0 = 11 \quad \therefore X_1 = 10$

$\therefore X = \begin{bmatrix} 10 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Page 3